

باسمه تعالی

۱- معادلات زیر را حل کنید:

(از روش تفکیک متغیرها)

$$a) \quad \frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} \quad 0 \leq x \leq 2, \quad t \geq 0$$

$$u(x, 0) = x + 1$$

$$\left. \frac{\partial u}{\partial t} \right|_{t=0} = \sin x$$

$$u(0, t) = 1$$

$$u(2, t) = 3$$

(از روش تفکیک متغیرها)

$$b) \quad \frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2} \quad 0 \leq x \leq 2, \quad t \geq 0$$

$$u(x, 0) = x$$

$$u(0, t) = 2$$

$$u(2, t) = 6$$

(از روش دالامبر)

$$c) \quad 4 \frac{\partial^2 u}{\partial t^2} - 25 \frac{\partial^2 u}{\partial x^2} = 0$$

$$u(x, 0) = \sin 2x$$

$$\left. \frac{\partial u}{\partial t} \right|_{t=0} = 0$$

(از روش دالامبر)

$$c) \quad 20 \frac{\partial^2 u}{\partial x^2} + 19 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = x^2 + y$$

(از روش دالامبر)

$$d) \quad \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = 0$$

$$z(x, x) = e^{3x}$$

$$z(x, -2x) = \cos 3x$$

$$e) \quad x \frac{\partial u}{\partial x} + (y - 1) \frac{\partial u}{\partial y} = u - 2$$

$$u(x, 3) = x - 2$$

(از روش دالامبر)

$$f) \quad \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = \frac{1}{2}$$

$$g) \quad \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = 1$$

$$u(x, x) = \frac{x}{2}$$

(از روش تفکیک متغیر ها)

$$h) \quad y \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial u}{\partial x} = x^2$$

$$i) \quad 2 \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} = 2x$$

$$u(0, y) = \sqrt{y}$$

(از روش تفکیک متغیر ها)

$$j) \quad 4 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 \leq x \leq 4, \quad t \geq 0$$

$$u(x, 0) = x$$

$$u(0, t) = 10$$

$$\frac{\partial u}{\partial x}(4, t) = 0$$

$$k) \quad x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$$

$$z(x, 2) = x - 1$$

(از روش تفکیک متغیر ها)

$$l) \quad \frac{\partial^2 u}{\partial x^2} = 4 \frac{\partial u}{\partial y}$$

$$u(0, y) = 8e^{-3y}$$

(از روش دالامبر به دو روش اپراتور تجزیه شدنی و نشدنی)

$$m) \quad \frac{\partial^2 u}{\partial x^2} - 3 \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial^2 u}{\partial y^2} = xsiny$$

(از روش دالامبر به دو روش اپراتور تجزیه شدنی و نشدنی)

$$n) \quad \frac{\partial^2 u}{\partial x^2} - 5 \frac{\partial^2 u}{\partial x \partial y} + 6 \frac{\partial^2 u}{\partial y^2} = x^2 y$$

(از روش دالامبر به دو روش اپراتور تجزیه شدنی و نشدنی)

$$o) \quad \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = ye^x$$

(از روش دالامبر)

$$p) \quad \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = t$$

$$u(x, 0) = x$$

$$\frac{\partial u}{\partial t}(x, 0) = 2$$

$$u(0, t) = 2t$$

$$u(1, t) = t$$